

*Amendments to the Specification*

Please substitute the attached replacement sheet for the "Abstract" section.

Page 1, please modify the two paragraphs at lines 16-19 as follows:

Apparatus and Method for Trigonometric Interpolation," ~~Attorney Docket No.~~  
1904.0140001 U.S. Patent Application Serial No. 09/699,088, filed October 30, 2000;  
and

B<sup>1</sup> Apparatus and Method for Rectangular-to-Polar Conversion," ~~Attorney~~  
~~Docket No. 1904.0140003~~ U.S. Patent Application Serial No. 09/698,249, filed  
October 30, 2000.

Page 3, please modify the paragraph beginning at line 19 as follows:

B<sup>2</sup> For proper operation, these third generation systems require proper  
synchronization between the transmitter and the receiver. More specifically, the  
frequency and phase of the receiver local oscillator should substantially match that of  
the transmitter local oscillator. When there is a mismatch, then an undesirable  
rotation of the symbol constellation will occur at the receiver, which will seriously  
degrade system performance. When the carrier frequency offset is much smaller than  
the symbol rate, the phase and frequency mismatches can be corrected at baseband by  
using a phase rotator. It is also necessary to synchronize the sampling clock such that  
it extracts symbols at the correct times. This can be achieved digitally by performing  
appropriate digital ~~resamples~~ resampling.

Page 6, please modify the paragraph at line 24 as follows:

B3

FIG. 3 illustrates [a] Lagrange basis polynomials.

Page 7, please modify the paragraph beginning at line 24 as follows:

B4

FIG. 16A-D illustrates a comparison of the amount of interpolation error using (A) ~~Language~~ Lagrange cubic, (B) the trigonometric interpolator 1000, (C) the trigonometric interpolator 1400, (D) the optimal structure (to be discussed in Section 4).

Page 9, please modify the paragraph beginning at line 6 as follows:

B5

FIG. 33 illustrates a signal with two samples/symbol and 40% excess bandwidth according to embodiments of the present invention.

Page 10, please modify the six paragraphs beginning at line 9 as follows:

B6

FIG. 49 illustrates a Booth multiplier ~~according to embodiments of the present invention.~~

FIG 50 illustrates an original Booth table 5000.

FIG. 51 illustrates ~~an~~ a negating ~~booth~~ Booth table 5100 according to embodiments of the present invention.

FIG 52 illustrates [an] a negating Booth multiplier 5200.

FIG. 53 illustrates a conditionally negating Booth decoder 5300 according to embodiments of the present invention.

B6  
FIG. 54 illustrates a conditionally negating multiplier 5400 according to  
embodiments of the present invention.

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Page 11, please modify the two paragraphs beginning at line 6 as follows:

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B7  
FIG. 64[.] illustrates simultaneous operation of a symbol-timing synchronizer  
and a carrier phase recovery system 6400 according to embodiments of the present  
invention.

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FIG. 65A-65B illustrate a flowchart ~~6200~~ 6500 associated with the  
synchronizer ~~6100~~ 6400 according to embodiments of the present invention.

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Page 11, please modify the paragraph beginning at line 23 as follows:

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B8  
FIGs. 75A-B illustrate impulse responses of the non-center-interval  
interpolation filter (A) before and ~~(b)~~ (B) after optimization, according to  
embodiments of the present invention.

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Page 29, please modify the paragraph beginning at line 22 as follows:

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B9  
FIG. 10 illustrates [an] a trigonometric interpolator 1000 that is one circuit  
configuration that implements the trigonometric interpolator equations (2.9)-(2.11),  
where the number of data samples is  $N=4$ . The interpolator 1000 is not meant to be  
limiting, as those skilled in the arts may recognize other circuit configurations that  
implement the equations ~~(2.8)~~ (2.9) - (2.11). These other circuit configurations are  
within the scope and spirit of the present invention.

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Page 30, please modify the paragraph beginning at line 6 as follows:

B10 The adder/subtractor module 1006 includes multiple adders (or subtractors)

1014, where subtraction is indicated by a (-) ~~signs~~ sign.

Page 30, please modify the paragraph beginning at line 19 as follows:

B11 In step 1706, the adder/subtractor module 1006 generates one or more trigonometric coefficients according to the equation (2.9). In FIG. 10, the coefficients are represented by  $C_0$ ,  $C_1$ , and  $C_2$  for  $N=4$ , where the ~~coefficients~~ coefficient  $C_1$  is a complex coefficient ~~coefficients~~.

Page 31, please modify the paragraph beginning at line 11 as follows:

B12 The trigonometric interpolator is not limited to the 4<sup>th</sup> degree embodiment that is shown in FIG. 10. The trigonometric interpolator can be configured as an  $N^{\text{th}}$  degree interpolator based on  $N_{\text{data}}$  points, as represented by the equations (2.9)-(2.11). These other  $N^{\text{th}}$  degree interpolators are within the scope and spirit of the present invention. For example and without limitation, FIG. 11 illustrates an ~~interpolator~~ interpolator 1100 having  $N=8$ . The trigonometric interpolator 1100 includes: a delay module 1102, an adder/subtractor module 1104 (having two scaling multipliers having coefficients  $\cos(\pi/4)$ ), an angle rotator module 1106, and an adder 1108\_(having [an] a 1/8 scale factor that is not shown). The operation of the

B12  
interpolator 1100 will be understood by those skilled in the arts based on the discussion herein.

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Page 33, please amend the paragraph beginning at line 13 as follows:

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B13  
The critical path of the Farrow structure 400 (FIG. [2] 4) ~~are~~ is now compared to that of the trigonometric interpolator. The Farrow structure implements the Lagrange interpolator as discussed above. The Farrow structure 400 is shown in FIG. 9 (or FIG. 4), with the critical path 902 indicated. The critical path 902 for this structure includes one scaling multiplier 904 and  $N - 1$  data multipliers 906.

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Page 37, please amend the two paragraphs beginning at line 7 as follows:

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B14  
In step 1908, the adder/subtractor module 1402 generates one or more trigonometric coefficients according to modifications to the equation ([2.8]2.9). In the  $N=4$  case, equations (2.25) are implemented by the module 1402. In FIG. 14, for  $N=4$ , the coefficients are represented by  $C_0$  and  $C_1$ , where the coefficient  $C_1$  is a complex coefficient ~~coefficients~~. By comparing with FIG. 10, it is noted that the  $C_2$  coefficient is 0. Additionally, the adder/subtractor module 1402 outputs the  $K$  value for further processing. Notice also that in FIG. 14, the output scaling factor has been changed from  $\frac{1}{4}$  to  $\frac{1}{2}$ . This reflects several other straightforward ~~simplifications~~ simplifications that have been made to module 1402 and in the angle rotator 1010b. In embodiments, the steps 1906 and 1908 are to be performed ~~performed~~

simultaneously by the adder/subtractor module 1402, as will be understood by those skilled in the relevant arts.

B14  
In step 1910, the angle rotator 1010b rotates the complex coefficient  $C_1$  in a complex plane according the offset  $\mu$ , resulting in a rotated complex coefficient. In embodiments, as discussed herein, the angle rotator[s] 1010b is table look-up. In which case, a complex rotation factor is ~~retrieve~~ retrieved from the table lookup based on the offset  $\mu$ , and the resulting rotation factor is then multiplied by the corresponding complex coefficient, to generate the respective rotated complex coefficient. The rotation factor includes the evaluation of the cosine and sine factors that are shown in equations (2.21). Note that since  $C_2 = 0$ , the angle rotator 1010a is replaced with the multiplier 1404.

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Page 38, please amend the paragraph beginning at line 6 as follows:

B15  
The simplified trigonometric interpolator is not limited to the four-sample embodiment that is shown in FIG. 14. The simplified trigonometric interpolator can be configured as an N-sample interpolator based on  $N[-]_{\text{data}}$  points, as represented by the equations (2.28)-(2.30). These other N-sample interpolators are within the scope and spirit of the present invention. For example and without limitation, an interpolator with  $N=8$  is discussed below.

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Page 39, please amend the paragraph beginning at line 5 as follows:

β16

How does the simplified interpolator 1400 (FIG. 14) perform as compared to the interpolator 1000 (FIG. 10)? FIGs. 16A-C show the frequency responses, in solid lines, of the Lagrange cubic interpolator 400 (FIG. [14]4), the interpolator 1000 (FIG.10) and the simplified interpolator 1400 (FIG. 14), respectively. For an input signal whose spectrum is a raised cosine with  $\alpha=0.4$ , as shown in dashed lines, the amount of interpolation error corresponds to the gray areas. Clearly, the interpolator 1400 produces less error than the Lagrange cubic interpolator 400 and the interpolator 1000. (FIG. 16D will be discussed in Section 4.)

Page 41, please amend the two paragraphs beginning at line 1 as follows:

β17

We can easily adapt the trigonometric interpolator described herein to efficiently create such a sampling rate conversion system, but one that does not require such filtering operations. If we denote the integer factor by which we desire to increase the data rate as  $L$  (in the above example,  $L = 4$ ) we proceed as follows. We build the system 7800 shown below in Fig.78[)]. System 7800 includes a Delay Module 7802 and Add/Subtract Module 7804 (that are similar to ~~that~~ those in FIG. 10), and such that it can accommodate incoming data at a rate  $r$ . We now build  $L$  copies of the Angle-Rotation Module 7806 (similar to that in FIG. 10), with each one being fed by the same outputs of the Add/Subtract Module. Within each of these  $L$  Angle-Rotation Modules 7806 we fix the  $\mu$  value; that is, each one has ~~one~~ has a different one of the values:  $1/L, 2/L, \dots, (1-L)/L$ . With such fixed  $\mu$  values, each Angle-Rotation Module 7806 can be constructed as a set of fixed multipliers (a very

special case of the table-lookup method), although any of the Angle-Rotation Module implementations previously discussed can be employed.

B17  
As shown in Fig. 78, the L-1 outputs, i.e., the interpolated samples that are offset by the values  $1/L, 2/L, \dots, (L-1)/L$  from the first of the two data points (indicated as  $\mu = 0$  and  $\mu = 1$  in the Delay Module of Fig. [A] 78) are routed to a multiplexer 7808, along with the input data point from which all interpolated samples are offset. The multiplexer 7808 simply selects these samples, in sequence, and provides them to the output at the expanded data rate  $L \times r$ .

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Page 42, please amend the paragraph beginning at line 2 as follows:

B18  
In this Section we have described an interpolation method that we have devised that uses trigonometric series for interpolation. Comparing the interpolations using the trigonometric polynomial and the Lagrange polynomial of the same degree, the trigonometric-based method achieves higher interpolation accuracy while simultaneously reducing the computation time and the amount of required hardware. Moreover, the trigonometric-based method ~~performs~~ performs operations that are similar to ~~that those~~ of a phase rotator for carrier phase ~~recovery~~ adjustment. This allows a reduction in the overall synchronization circuit complexity by sharing resources.

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Page 45, please amend the paragraph starting at line 19 and continuing onto page 46, as follows:



Since  $w$  is a rectangular function,  $W$  must be a sinc function 2210. Convolving  $F_c$  and the sinc function  $W$  simply interpolates the frequency samples  $\hat{F}(k)$  to obtain  $F(\Omega)$ ,  $-\infty < \Omega < \infty$ , shown as response 2212. (Here we have plotted the symmetric  $F$  only on the positive half of the  $\Omega$  axis.) We thus have

$$\hat{F}(k) = F(\Omega)|_{\Omega=2\pi k/N}, \quad -\frac{N}{2} \leq k \leq \frac{N}{2}. \quad (3.8)$$

B19 From response 2212, the continuous-frequency response  $F(\Omega)$  is uniquely determined by an infinite number of equally spaced frequency samples  $\hat{F}(k)$ . If we modify the frequency samples 2214 near the passband edge to let the transition between the passband and stopband be more gradual, as depicted in FIG. 23, then the ripple is decreased. FIG. 23 demonstrates gradually reduced samples 2302, and the reduction of ripples in the overall response 2304, as compared to the response 2212 in FIG. 22. The cost of this improvement is[,] an increased transition bandwidth in the response 2304, as compared to the response 2212.

Page 49, line 9, please replace Equation 3.21 with the following to correct spacing problems:

B20

$$\begin{aligned} F(\Omega) &= \left( \sum_{k=-M}^M \hat{F}(k) \delta\left(\Omega - \frac{2\pi}{N}k\right) \right) \otimes \text{sinc}(\Omega) \\ &= \sum_{k=-M}^M \hat{F}(k) \text{sinc}\left(\Omega - \frac{2\pi}{N}k\right). \end{aligned} \quad (3.21)$$

Page 51, please replace equation 4.3 with the following to delete the minus sign:

$$F_d(\omega, \mu) = e^{[(-)]j\omega\mu}$$

Page 59, please amend the paragraph beginning at line 21 as follows:

- b. Real  $\hat{F}_\mu(1)$  values are used. The output of the angle-rotator  ~~$\text{Re}(cie^{i\frac{4}{3}\mu})$~~   
 $\text{Re}(c_1 e^{i\frac{4}{3}\mu})$  is multiplied by  $\hat{F}_\mu(1)$ . Thus, one more real multiplexer is needed.

Page 68, please amend the two paragraphs starting at line 10 as follows:

The fine adjustment circuit 3804 generates a fine adjust value  $(1 - \frac{1}{2}\theta_L^2)$ ,  
 where  $\theta_L$  is the least significant word of the input angle  $\theta$ .

The second butterfly circuit 3810 multiplies the output of circuit 3806 by  $\theta_L^+$   
 and the fine adjustment value from circuit 3804, to perform a fine rotation that results  
 in the rotated complex number 3814. The + on the  $\theta_L^+$  denotes ~~the~~ that an error value  
 $\Delta_{\sin\theta_L}$  has been added to improve the accuracy of the fine rotation.

Page 79, please amend the two paragraphs starting at line 12 as follows:

In step 4110, a fine adjustment circuit 3904 generates a fine adjust value  
 $\left(\frac{\delta_{[\cos\theta_1]} - \theta_l^2}{\delta_{[\cos\theta_1]} - \frac{1}{2}\theta_l^2}\right)$  based on the  $\theta_l$  angle and  $\delta_{[\cos\theta_1]}$ .

In step 4112, the butterfly circuit 3910 multiplies the intermediate complex  
 signal by the  $\theta_l$  angle, and the fine adjustment value  $\left(\frac{\delta_{[\cos\theta_1]} - \theta_l^2}{\delta_{[\cos\theta_1]} - \frac{1}{2}\theta_l^2}\right)$  to

B24  
perform a fine rotation of the intermediate complex number, resulting in the output complex signal 3912.

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Page 83, please amend the paragraph beginning at line 24 as follows:

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B25  
For our structure, we chose the internal wordlengths and multiplier sizes as indicated in FIG. 42. The phase-accumulator that generates  $[\theta] \bar{\theta}$  as well as the circuit that maps an angle in the range  $[0, 2\pi]$  into  $[0, \pi/4]$ , are described in (Madisetti, A., "VLSI architectures and IC implementation for bandwidth efficient communications," Ph.D. dissertation, University of California, Los Angeles (1996)). These structures are also employed here in our test. Truncating the 32-bit phase word to 14 bits, this structure has achieved a SFDR of 90.36 dB, as shown in FIG. 43. This is 6 dB better than the single stage method.

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Page 85, please replace equation 5.68 at line 5 with the following to change the "-" sign in the second equation to a "+" sign, an obvious error:

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B26

$$\begin{aligned} X_1 &= X_0 - Y_0 \tan \theta_M \\ Y_1 &= Y_0 + X_0 \tan \theta_M \end{aligned} \quad (5.68)$$

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Page 89, please amend the two paragraphs beginning at line 23 as follows:

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In step 4510, a fine adjustment circuit 4404 generates a fine adjust value

B27  
 $(\delta_{[\cos \theta_l]} - \theta_l^2) (\delta_{[\cos \theta_l]} - \frac{1}{2} \theta_l^2)$  based on the  $\theta_l$  angle and  $\delta_{\cos \theta_m}$ .

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B27 In step 4512, the butterfly circuit 4410 multiplies the intermediate complex signal by the  $\theta_i$  angle, and the fine adjustment value  $\left( \delta_{[\cos \theta_i]} - \theta_i^2 \right) \left( \delta_{[\cos \theta_i]} - \frac{1}{2} \theta_i^2 \right)$  to perform a fine rotation of the intermediate complex signal, resulting in the output complex signal. X

Page 98, please amend the section heading at line 5 as follows:

B28

**5.8.2.3 How to make a Conditionally Negative Negating  
Booth Multiplier**

Page 133, please amend the paragraph beginning at line 17 as follows:

B29 In an embodiment where the invention is ~~implement~~ implemented using software, the software may be stored in a computer program product and loaded into computer system 7702 using removable storage drive 7714, hard drive 7712 or communications interface 7722. The control logic (software), when executed by the processor 7704, causes the processor 7704 to perform the functions of the invention as described herein.

Page 136, please amend equation B.2 at line 12 as follows to change m to N as shown, correcting an obvious error:

B30

$$c_k = \sum_{m=-N/2+1}^{N/2} \tilde{y}(m) W_{[m]N}^{km} = \sum_{m=-N/2+1}^{N/2} (y(m) + mK) W_N^{km} \quad (\text{B.2})$$

Page 136, please replace equation B.3 with the following:

15B31

$$\begin{aligned}
 c_k &= \sum_{m=-N/2+1}^{N/2} y(m) W_N^{km} - \sum_{m=-N/2+1}^{N/2} \left( m \frac{2}{N} \sum_{n=-N/2+1}^{N/2} (-1)^n y(n) \right) W_N^{km} \\
 &= \sum_{m=-N/2+1}^{N/2} y(m) W_N^{km} - \sum_{n=-N/2+1}^{N/2} \left( \sum_{m=-N/2+1}^{N/2} m \frac{2}{N} W_N^{km} \right) (-1)^n y(n) \quad (\text{B.3 (B.3)}) \\
 &= \sum_{m=-N/2+1}^{N/2} y(m) W_N^{km} - \sum_{m=-N/2+1}^{N/2} \left( \sum_{n=-N/2+1}^{N/2} n \frac{2}{N} W_N^{kn} \right) (-1)^m y(m)
 \end{aligned}$$